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# THEORETICAL MODELING OF THE NUCLEATION OF INTERNAL LATENT THERMAL DEFECTS IN A BITUMINOUS MEDIUM WITH RUBBER INCLUSIONS

# ТЕОРЕТИЧНЕ МОДЕЛЮВАННЯ ЗАРОДЖЕННЯ ВНУТРІШНІХ ПРИХОВАНИХ ТЕРМІЧНИХ ДЕФЕКТІВ У БІТУМНОМУ СЕРЕДОВИЩІ З ГУМОВИМИ ВКЛЮЧЕННЯМИ

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**Abstract.** The current state and progress of the technology and science associated with the reuse and recycling of the tyre rubber worldwide in the road industry compels to study more thoroughly high and low temperature performance of the road bitumen modified with rubber crumbs, permitting to understand influence of the temperature, rubber grain size and mixture bitumen-rubber modification on the composite strength and sustainability. Below, these issues are studied taking into account the peculiarities of the thermomechanical properties of rubber associated with its low rigidity when changing shape, practical incompressibility when changing volume, and low (zero or even negative) coefficient of linear thermal expansion.

The purpose of the study is to determine the reasons leading to a violation of the strength of asphalt concrete materials with admixtures of rubber crumb. For this purpose, the influence of the incompatibility of thermomechanical characteristics (moduli of elasticity, Poisson's ratios and coefficients of thermal expansion) of bitumen and rubber on the concentration of additional internal thermal stresses in the system caused by seasonal and daily temperature changes is analyzed.

Using the relations of the theory of thermoelasticity, a mathematical model of thermal deformation of crumb rubber in a bitumen medium has been constructed. With the possibility of complete and surface modification of rubber with bitumen, solutions for three-phase media are constructed, which make it possible to trace the influence of the parameters of each phase on the thermal stress fields in the system. It has been established that additional thermal stresses in bitumen, due to the thermomechanical incompatibility of the physical parameters of the phases, are concentrated in the zone of its contact with the surface of the rubber crumb and can cause defects and chippings in it. The influence of the effect of modifying rubber crumb with bitumen and of the depth of its penetration into crumb of different sizes on reducing thermal stresses in the system and increasing its sustainability is considered.

**Keywords:** asphalt concrete material, rubber inclusions, incompressibility of rubber, thermal stress concentrators, modified rubber.

## Introduction

At present, the problem of rubber production waste disposal, in particular, rubber tires of cars, is becoming more and more acute in the world, since during their burial and open storage, air, wastewater and soil are polluted with harmful and toxic products of their decay [1,2,3,4,5,6]. One of the trends in this direction is associated with the possibility of using crushed rubber as an additive in asphalt concrete pavement material [7,8,9]. The first experiments to create such materials were carried out long ago. Pilot sections of roads and airfields with crumb rubber have been built in various countries and regions. At first, they showed rather high characteristics, their increased crack resistance, water resistance, a decrease in the level of vibrations and noise on them, a decrease in braking distance, etc. However later, this material began to show itself from the negative side, inasmuch as such mix proved to be exposed to fast ageing, disintegration and destruction. As this took place, rubber crumbs, not bound to bitumen by strong bonds, crumbled from the coating and were carried by the wind almost unchanged, polluting the surroundings.

To eliminate this drawback, the crumb began to be modified before introducing it into bitumen to form a developed interfacial layer at the “rubber crumb – bitumen” phase boundary and increase the adhesion of the binder to the mineral components of asphalt concrete [10,11,12,13]. The bitumen and chips modified in this way are a homogeneous mixture of oxidized road bitumen with fairly fine crumbs from general-purpose rubbers subjected to special chemical treatment during the manufacturing process. In this case, the rubber particles do not completely decompose and do not dissolve, but are associated with the bitumen components by strong, but rather flexible chemical bonds and show their qualities even in the composition of a new material. In so doing, the grains sizes are no more than 1 mm, and by volume, they usually occupy 5% - 7% of the total volume of the mixture. At the same time, attempts to increase the grain size (up to 1 cm or more) and the volume percentage of the mixture (up to 20% or more) usually led to a negative result. Apparently, this is due to the special thermomechanical properties that rubber has (as a hyperelastic material) and the role played by a thin interfacial layer on the surfaces of small and large grains of inclusions when the temperature of the system changes [14, 15,16].

Note that rubbers usually have a low modulus of elasticity ( $E = 1 \div 10\text{МПа}$ ), but a very large Poisson's ratio. So, for most unvulcanized rubbers, it is equal to $\upsilon = 0.5$, for vulcanized rubbers, it is $\upsilon = 0.48 \div 0.5$. Therefore, during deformations of the rubber body associated with a change in its shape, rubber is characterized by very low rigidity and high elasticity. This property manifests itself under uniaxial and biaxial force loading of rubber bodies, as well as during their shear deformations.

When trying to deform rubber with a change in its volume, but with the preservation of its shape, it is characterized by great rigidity, since $\upsilon = 0.5$. Then, its volumetric module of elasticity $K = E \big/ \left(3\left(1-2\upsilon\right)\right) = \infty$ and this material becomes volumetrically absolutely rigid. Therefore, from an engineering point of view, rubber is considered as an incompressible material [17]. This property is typical for its three-dimensional thermally stressed deformation under temperature changes in a limited volume and triaxial force effects, leading to all-round compression. As shown below, the incompressibility of rubber does not allow it to elastically compensate for thermal deformations of bitumen, due to which additional thermal stresses arise in it. Owing to the fact that an interfacial compressible layer of small thickness is formed on the crumb surface upon contact with bitumen, it relieves thermal stresses to some extent due to its own compliance at small crumb sizes. On large inclusions, a thin pliable layer cannot compensate for their rigidity and large thermal stresses are generated on their surfaces, which lead to local plastic deformations, defects and cracks in bitumen.

To these characteristics of rubber, one more feature must be added. The fact is that even at moderate temperatures, its coefficient of thermal expansion varies over a wide range, taking zero and even negative values. At the same time, at high temperatures ($150^{\circ}$ C and above), its coefficient of thermal expansion due to the formation of vapor-filled pores in it is an order of magnitude greater than the values of this coefficient of other materials [18, 19]. Such incompatibility of thermomechanical parameters of bitumen and rubber is an additional factor contributing to the concentration of thermal stress on the contact surfaces of bitumen with large rubber inclusions and their delamination. To study these effects, based on the theory of thermoelasticity, a mathematical model of the thermally stressed state of an asphalt concrete medium with spherical rubber inclusions under the action of temperature disturbances was developed, constitutive differential equations were formulated, and their closed-form solutions were found. An analysis of the constructed thermal stress fields made it possible to establish the main features of the origin of additional latent microdefects in a composite material reinforced with rubber crumbs.

When evaluating the constructed solutions, it must be taken into account that they are obtained by using the methods of the theory of elasticity, while bitumen and rubber are viscoelastic materials. Therefore, these solutions, although they reflect the obtained features of deformation of their combination, should be refined when calculating real road coatings. Such solutions improvement can be performed using the principle of correspondence of solutions to the problems dealing with the theories of elasticity and viscoelasticity, described in the works by L. Khazanovich [20] and G.H. Paulino, Z.-H.Jin [21].

Thermally stressed state of asphalt concrete material with spherical rubber inclusion

To establish the general peculiarities of distribution of thermal stress fields in a bitumen medium with rubber inclusions, we will use the model of a spherical elastic body 1 of radius $r_1$, bound on its surface $r = r_1$ with elastic medium 2 (Fig. 1), for which the radius $r_2$ on the outer boundary spherical surface can take infinitely large values.

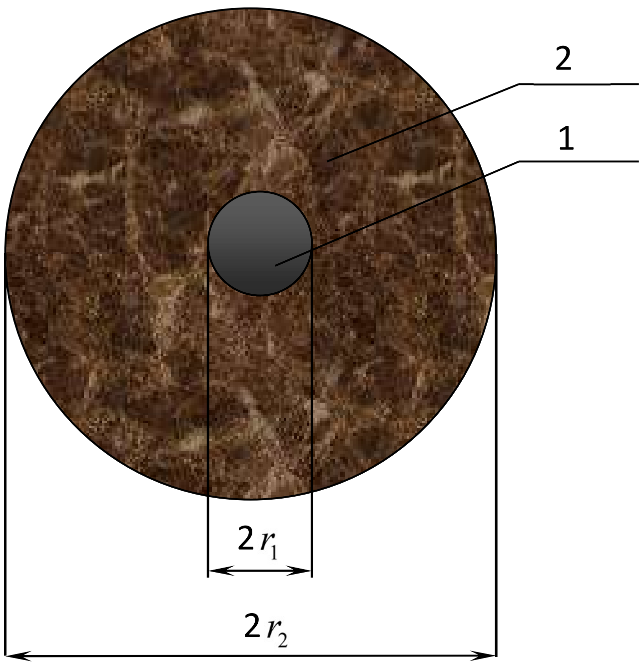


***Figure 1*** – Diagram of a spherical elastic inclusion 1 in an elastic medium 2

***Рисунок 1*** – Схема сферичного пружного включення 1 у пружному середовищі 2

Let us consider the thermoelastic equilibrium of the system, when its temperature at all points changes stationary from 0 to $T$. Let the thermomechanical properties of body 1 and medium 2 be characterized by the values of their moduli of elasticity $E_1$ and $E_2$, Poisson's ratios $\upsilon_1$ and $\upsilon_2$ and coefficients of their linear thermal expansion $\alpha_1$ and $\alpha_2$. We will describe the thermal deformation of the system in the spherical coordinate system $Or\varphi\theta$, origin $O$ of which coincides with the center of body 1 (Fig. 2).

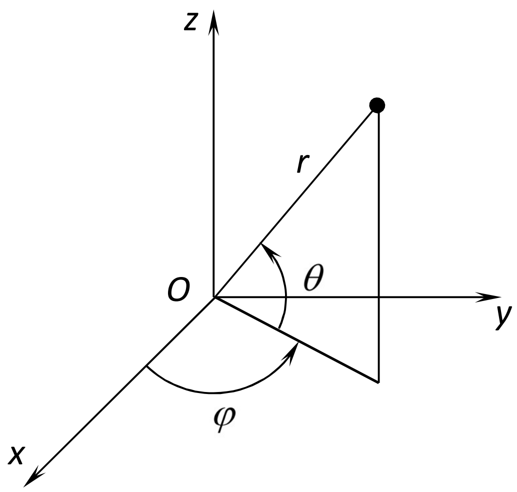


***Figure 2*** – Diagram of a spherical coordinate system

***Рисунок 2*** – Діаграма сферичної системи координат

The deformed state of the elastic system in these coordinates has the property of central symmetry, so its equilibrium is described by two equations [22,23,24,25]

$$\frac{d\sigma_r^{(i)}}{dr}+\frac{2\sigma_r^{(i)}-\sigma_\varphi^{(i)}-\sigma_\theta^{(i)}}{r}=0 \quad \left(i=1,2\right), \tag{1}$$

where $\sigma_r^{(i)}, \sigma_\varphi^{(i)}, \sigma_\theta^{(i)}$ are the normal stresses on the corresponding elementary areas $r=const$, $\varphi=const$, $\theta=const$ in body 1 at $i=1$ and in medium 2 at $i=2$. They are expressed in terms of the components of the relative strain tensor $\varepsilon_r^{(i)}$, $\varepsilon_\varphi^{(i)}=\varepsilon_\theta^{(i)}$:

$$\begin{gathered}\sigma_r^{(i)}=\frac{E_i\left(1-\upsilon_i\right)}{\left(1+\upsilon_i\right)\left(1-2\upsilon_i\right)}\varepsilon_r^{(i)}+\frac{2E_i\upsilon_i}{\left(1+\upsilon_i\right)\left(1-2\upsilon_i\right)}\varepsilon_\varphi^{(i)}-\frac{E_i}{\left(1-2\upsilon_i\right)}\alpha_i T,\\ \sigma_\varphi^{(i)}=\sigma_\theta^{(i)}=\frac{E_i\upsilon_i}{\left(1+\upsilon_i\right)\left(1-2\upsilon_i\right)}\varepsilon_r^{(i)}+\frac{E_i}{\left(1+\upsilon_i\right)\left(1-2\upsilon_i\right)}\varepsilon_\varphi^{(i)}-\frac{E_i}{\left(1-2\upsilon_i\right)}\alpha_i T \quad \left(i=1,2\right)\end{gathered} \tag{2}$$

It must be emphasized here that on the right-hand sides of these expressions, all terms in the denominators contain the factor $\left(1-2\upsilon_i\right)$. Since for rubber $\upsilon\approx 0.5$, all these terms are divisible by a small value, which already at this level introduces a peculiar specificity of the thermomechanical behavior of rubber.

The strains used in Eqs. (2) are determined through the radial displacement $u^{(i)}$

$$\varepsilon_r^{(i)}=\frac{du^{(i)}}{dr}, \qquad \varepsilon_\varphi^{(i)}=\frac{u^{(i)}}{r}, \qquad \left(i=1,2\right). \tag{3}$$

Using these equalities, we reduce Eqs. (2) to the form

$$\sigma_r^{(i)}(r)=\frac{E_i(1-\upsilon_i)}{(1+\upsilon_i)(1-2\upsilon_i)}\frac{du^{(i)}}{dr}+\frac{2E_i\upsilon_i}{(1+\upsilon_i)(1-2\upsilon_i)}\frac{u^{(i)}}{r}-\frac{E_i}{(1-2\upsilon_i)}\alpha_i T,$$
$$\sigma_\varphi^{(i)}=\sigma_\theta^{(i)}=\frac{E_i\upsilon_i}{(1+\upsilon_i)(1-2\upsilon_i)}\frac{du^{(i)}}{dr}+\frac{E_i}{(1+\upsilon_i)(1-2\upsilon_i)}\frac{u^{(i)}}{r}-\frac{E_i}{(1-2\upsilon_i)}\alpha_i T \quad (i=1,2). \tag{4}$$

After substituting Eqs. (4) into Eqs. (1), we obtain the equations

$$\frac{d^2u^{(i)}}{dr^2}+\frac{2}{r}\frac{du^{(i)}}{dr}-\frac{2}{r^2}u^{(i)} \quad (i=1,2). \tag{5}$$

Let us bring them to a form convenient for integration

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r^2u^{(i)}\right)\right]=0 \quad (i=1,2). \tag{6}$$

System of Eqs. (6) has a solution

$$u^{(1)}(r)=rC_1^{(1)}+\frac{1}{r^2}C_1^{(2)} \quad (r\le r_1),$$
$$u^{(2)}(r)=rC_1^{(2)}+\frac{1}{r^2}C_2^{(2)} \quad (r\ge r_1), \tag{7}$$

where $C_1^{(1)}$, $C_2^{(1)}$, $C_1^{(2)}$, $C_2^{(2)}$ are the constants that are determined from the boundary conditions at $r=0$ and $r_2\to\infty$ and conjugation conditions for solutions (7) on the surface $r=r_1$. These conditions look like:

$$u^{(1)}(0)=0, \tag{8}$$

$$u^{(1)}(r_1)=u^{(2)}(r_1), \tag{9}$$

$$\sigma_r^{(1)}(r_1)=\sigma_r^{(2)}(r_1), \tag{10}$$

$$\sigma_r^{(2)}(r)\to 0 \text{ at } r_2\to\infty. \tag{11}$$

It follows from Eq. (8) that the existence of function $u^{(1)}(r)$ in Eqs. (7) is possible only when

$$C_2^{(1)}=0. \tag{12}$$

Using Eqs. (4) and (7), we represent the functions $\sigma_r^{(i)}(r)$, $\sigma_\varphi^{(i)}(r)$, $\sigma_\theta^{(i)}(r)$, presented in Eqs. (10) and (11), in the form:

$$\sigma_r^{(i)}(r)=\frac{E_i}{(1-2\upsilon_i)}C_1^{(i)}-\frac{2E_i}{r^3(1+\upsilon_i)}C_2^{(i)}-\frac{E_i}{(1-2\upsilon_i)}\alpha_i T,$$
$$\sigma_\varphi^{(i)}=\sigma_\theta^{(i)}=\frac{E_i}{(1-2\upsilon_i)}C_1^{(i)}+\frac{E_i}{r^3(1+\upsilon_i)}C_2^{(i)}-\frac{E_i}{(1-2\upsilon_i)}\alpha_i T \quad (i=1,2). \tag{13}$$

Then, to fulfill conditions (11), it is necessary that the sum of the first and third terms on the right side of expression $\sigma_r^{(i)}(r)$ with $i=2$ be equal to zero. This is possible with

$$C_1^{(2)}=\alpha_2 T. \tag{14}$$

Constants $C_1^{(1)}$ and $C_2^{(2)}$ are found from conditions (9) and (10). Let's rewrite them in the form:

$$r_1C_1^{(1)}=\frac{1}{r_1^{(2)}}C_2^{(2)}+r_1\alpha_2 T,$$
$$\frac{E_1}{(1-2\upsilon_1)}C_1^{(1)}-\frac{E_1}{(1-2\upsilon_1)}\alpha_1 T=-\frac{2E_2}{r_1^3(1+\upsilon_2)}C_2^{(2)}-\frac{E_2}{(1-2\upsilon_2)}\alpha_2 T. \tag{15}$$

From this system, it follows:

$$C_1^{(1)} = \frac{E_1(1+\upsilon_2)\alpha_1 T + 2E_2(1-2\upsilon_1)\alpha_2 T}{E_1(1+\upsilon_2)+2E_2(1-2\upsilon_1)},$$
$$C_2^{(2)} = \frac{r_1^3 E_1(1+\upsilon_2)(\alpha_1-\alpha_2)T}{E_1(1+\upsilon_2)+2E_2(1-2\upsilon_1)}. \quad (16)$$

With the help of Eqs. (12), (13), (16) with $i=1$, find thermal stresses $\sigma_r^{(1)}(r)$, $\sigma_\varphi^{(1)}(r)$, $\sigma_\theta^{(1)}(r)$ in inclusion 1

$$\sigma_r^{(1)}(r) = \sigma_\varphi^{(1)}(r) = \sigma_\theta^{(1)}(r) = \frac{2E_1E_2(\alpha_2-\alpha_1)T}{E_1(1+\upsilon_2)+2E_2(1-2\upsilon_1)} \quad (r \le r_1). \quad (17)$$

Radial thermal stresses $\sigma_r^{(2)}(r)$ in medium 2 are calculated using Eqs. (13), (14), (16) with $i=2$

$$\sigma_r^{(2)}(r) = \frac{2r_1^3}{r^3}\frac{E_1E_2(\alpha_2-\alpha_1)T}{E_1(1+\upsilon_2)+2E_2(1-2\upsilon_1)} \quad (r \ge r_1). \quad (18)$$

Thermal stresses $\sigma_\varphi^{(2)}(r)$, $\sigma_\theta^{(2)}(r)$ in medium 2 are calculated on the basis of Eqs. (4), (14), (16) with $i=2$

$$\sigma_\varphi^{(2)}(r) = \sigma_\theta^{(2)}(r) = -\frac{r_1^3}{r^3}\frac{E_1E_2(\alpha_2-\alpha_1)T}{E_1(1+\upsilon_2)+2E_2(1-2\upsilon_1)} \quad (r \ge r_1). \quad (19)$$

From Eqs. (17), (18), it follows that, in accordance with condition (10), the radial stresses $\sigma_r^{(1)}(r)$ in body 1 and $\sigma_r^{(2)}(r)$ in medium 2 on the contact surface $r=r_1$ are the same. In addition, in the medium 2, stress $\sigma_r^{(2)}(r)$ is twice the stresses $\sigma_\varphi^{(2)}(r)$, $\sigma_\theta^{(2)}(r)$ and they all decrease along the radial coordinate in proportion to the cube of the distance from the center of body 1 to the point in question. Note also that all stresses in the system are proportional to the difference $(\alpha_2-\alpha_1)$ and product $E_1E_2$.

Eqs. (17)-(19) are essentially simplified if $\upsilon = 0,5$. Then we have

$$\sigma_r^{(1)}(r) = \sigma_\varphi^{(1)}(r) = \sigma_\theta^{(1)}(r) = \frac{2E_2(\alpha_2-\alpha_1)T}{1+\upsilon_2} \quad (r \le r_1),$$
$$\sigma_r^{(2)}(r) = \frac{2r_1^3}{r^3}\frac{E_2(\alpha_2-\alpha_1)T}{1+\upsilon_2} \quad (r \ge r_1), \quad (20)$$
$$\sigma_\varphi^{(2)}(r) = \sigma_\theta^{(2)}(r) = -\frac{r_1^3}{r^3}\frac{E_2(\alpha_2-\alpha_1)T}{1+\upsilon_2} \quad (r \ge r_1).$$

In this case, the rubber inclusion behaves like an absolutely solid body and, therefore, thermal stresses (20) do not depend on its modulus of elasticity $E_1$.

The obtained Eqs. (17)-(19) and (20) allow us to establish the main features of the influence of additional thermal stresses in a bitumen medium with rubber inclusions on its thermal sustainability. At the same time, it is necessary again to pay attention to the significant thermomechanical incompatibility of rubber and bitumen. In the general case, their properties can vary over a wide range depending on their physical and mechanical structures. Below, in the analysis for rubber, we use the values $E_1 = 1 \div 10$ MPa, $\upsilon_1 = 0.48,\ 0.49$ and $0.5$, $\alpha_1 = 0$, although even at moderate temperatures $\alpha_1$ may be negative. For bitumen we accept $E_2 = 3000$ MPa, $\upsilon_2 = 0.25$, $\alpha_2 = 2.3 \cdot 10^{-4}$ 1/ °C.

With such values of the initial parameters and $T = 30^\circ\text{C}$, using Eqs. (17)-(19), we obtain $\sigma_r^{(1)}(r) = \sigma_\varphi^{(1)}(r) = \sigma_\theta^{(1)}(r) = (0.3414 \div 3.1245)$ MPa $(r \le r_1)$, $\sigma_r^{(2)}(r) = \frac{r_1^3}{r^3}(0.3414 \div 3.1245)$ MPa, $\sigma_\varphi^{(2)}(r) = \sigma_\theta^{(2)}(r) = -\frac{r_1^3}{r^3}(0.1707 \div 1.562)$ MPa $(r \ge r_1)$. Here, the first values in parentheses correspond to the case $E_1 = 1$ MPa, the second ones – to the case $E_1 = 10$ MPa.

Graphs of these functions for $E_1 = 10$ MPa are shown in Fig.3.

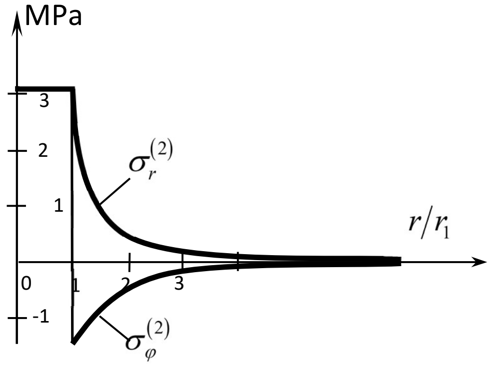


***Figure 3*** – Function distribution graphs $\sigma_r^{(i)}(r)$ and $\sigma_\varphi^{(i)}(r)$ $(i = 1, 2)$ in the rubber-bitumen system for the case $E_1 = 10$ MPa

***Рисунок 3*** – Графіки розподілу функцій $\sigma_r^{(i)}(r)$ і $\sigma_\varphi^{(i)}(r)$ $(i = 1, 2)$ в системі гума-бітум для випадку $E_1 = 10$ МПа

As can be seen, in a bituminous medium, the functions $\sigma_r^{(2)}(r)$, $\sigma_\varphi^{(2)}(r)$, $\sigma_\theta^{(2)}(r)$ are concentrated on the contact surface $r = r_1$ and decrease in inverse proportion to the cube of the coordinate $r$. At the same time, in the zone of its concentration, even at a low value $T$ and zero $\alpha_1$, these functions are in the zone of limit values for bitumen, which are $[\sigma_{\lim}] = 3 \div 8$ MPa with a time duration of 0.1 s. and temperature $T = 5 \div 10^\circ\text{C}$. In reality, with prolonged duration of temperature disturbances and large intervals of temperature change, the values $[\sigma_{\lim}]$ may be much lower.

An important issue is the study of the effect of rubber super rigidity, determined by the coefficient $\upsilon_1$, on the concentration of additional thermal stresses in bitumen. Table 1 shows the values of thermal stresses $\sigma_r^{(1)} = \sigma_\varphi^{(1)}$, $\sigma_r^{(2)}(r_1)$ and $\sigma_\varphi^{(2)}(r_1)$ at $\upsilon_1 = 0.48,\ 0.49$ and $0.5$.

It follows from these results that with increasing $\upsilon_1$ additional thermal stresses in bitumen increase rapidly, and at $\upsilon_1 = 0.5$, when the rubber becomes absolutely rigid, they greatly exceed its $[\sigma_{\lim}]$.

***Table 1*** – Values of maximum thermal stresses at various values of the coefficient $\upsilon_1$

***Таблиця 1*** – Значення максимальних термічних напружень при різних значеннях коефіцієнта $\upsilon_1$

| $E_1$, MPa | $\upsilon_1$ | $10^4\alpha_1$, $1/\ ^\circ C$ | $E_2$, MPa | $\upsilon_2$ | $10^4\alpha_2$, $1/\ ^\circ C$ | $\sigma_r^{(1)}=\sigma_\varphi^{(1)}$, MPa | $\sigma_r^{(2)}(r_1)$, MPa | $\sigma_\varphi^{(2)}(r_1)$, MPa |
|---|---|---|---|---|---|---|---|---|
| 10 | 0.48 | 0 | 3000 | 0.25 | 2.3 | 1.6396 | 1.6396 | – 0.8189 |
| 10 | 0.49 | 0 | 3000 | 0.25 | 2.3 | 3.1245 | 3.1245 | – 1.5623 |
| 10 | 0.50 | 0 | 3000 | 0.25 | 2.3 | 33.120 | 33.120 | – 16.560 |

As a result of the calculations performed, it can be assumed that in a small neighborhood of the rubber inclusion, bitumen (or asphalt concrete) is constantly in the critical or overcritical state, which leads to a decrease in its thermal sustainability, the formation of hidden local defects in it, and under the action of transport loads, to provoking delamination and chipping rubber inclusions. In this regard, a positive role is played by the practice of modifying bitumen with small rubber particles, on the surface of which, because of additional chemical treatment, an intermediate layer with enhanced mechanical properties is formed. It is obvious that this positive effect may not manifest itself on larger rubber inclusions. In this regard, the question of studying the thermally stressed state of a rubber inclusion with an additional elastic coating is of interest.

***Modeling the thermally stressed state of a rubber spherical inclusion with an elastic layer***

Consider the case when a spherical rubber inclusion 1, covered with a shell 2, is in a bituminous medium 3 (Fig.4).

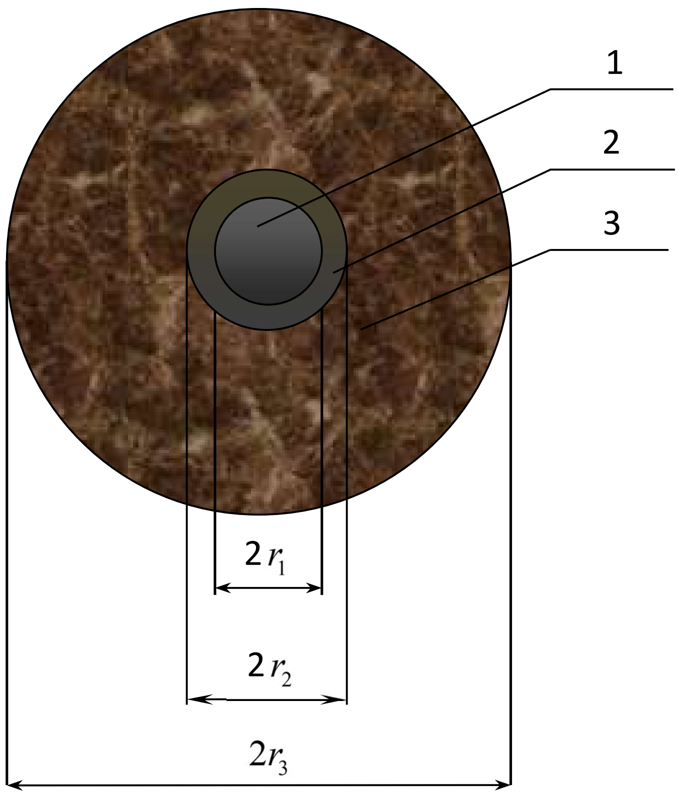


***Figure 4*** – Scheme of a spherical inclusion 1 with a spherical interlayer 2 placed in an elastic medium 3

***Рисунок 4*** – Схема сферичного включення 1 зі сферичною прошарком 2 у пружному середовищі 3

Body 1 is bounded by a spherical surface of radius $r_1$, layer 2 is limited by two surfaces of radii $r_1$ and $r_2$, medium 3 – by two surfaces of radii $r_2$ and $r_3$. It is assumed in the calculations that $r_3 \to \infty$. The thermomechanical properties of this system are characterized by the parameters $E_i$, $\upsilon_i$, $\alpha_i$ $\left(i=\overline{1,3}\right)$ at the same temperature $T$.

The thermally stressed equilibrium state of this three-phase system is described by three equations of the form (1)

$$\frac{d\sigma_r^{(i)}}{dr}+\frac{2\sigma_r^{(i)}-\sigma_\varphi^{(i)}-\sigma_\theta^{(i)}}{r}=0 \quad \left(i=\overline{1,3}\right), \tag{21}$$

in which, however, the index $i$ runs through the values 1,2,3 corresponding to the body numbers 1,2,3.

For the system under consideration, Eqs. (2) – (5) also retain their meaning for the values of the index $\left(i=\overline{1,3}\right)$ instead of $\left(i=1,2\right)$. These equations can be reduced to a system of three equilibrium equations expressed in terms of radial displacements $u^{(i)}$ $\left(i=\overline{1,3}\right)$

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r^2u^{(i)}\right)\right]=0 \quad \left(i=\overline{1,3}\right). \tag{22}$$

This system has a solution

$$\begin{aligned} u^{(1)}(r)&=rC_1^{(1)}+\frac{1}{r^2}C_2^{(1)} \quad (r\le r_1),\\ u^{(2)}(r)&=rC_1^{(2)}+\frac{1}{r^2}C_2^{(2)} \quad (r_1\le r\le r_2),\\ u^{(3)}(r)&=rC_1^{(3)}+\frac{1}{r^2}C_2^{(3)} \quad (r_1\le r\le r_2). \end{aligned} \tag{23}$$

Six constants $C_1^{(i)}$, $C_2^{(i)}$, presented in these equalities, are found from six boundary conditions for conjugation of functions (23) on contact surfaces

$$u^{(1)}(0)=0, \tag{24}$$

$$u^{(1)}(r_1)=u^{(2)}(r_1), \tag{25}$$

$$\sigma_r^{(1)}(r_1)=\sigma_r^{(2)}(r_1), \tag{26}$$

$$u^{(2)}(r_2)=u^{(3)}(r_2), \tag{27}$$

$$\sigma_r^{(2)}(r_2)=\sigma_r^{(3)}(r_2), \tag{28}$$

$$\sigma_r^{(3)}(r_3)=0 \quad r_2\to\infty, \tag{29}$$

From Eq. (24), it follows:

$$C_2^{(1)}=0. \tag{30}$$

We rewrite condition (29) in the form

$$\sigma_r^{(3)}(r_3)=\frac{E_3}{(1-2\upsilon_3)}C_1^{(3)}-\frac{2E_3}{r_3^3(1+\upsilon_3)}C_2^{(3)}-\frac{E_3}{(1-2\upsilon_3)}\alpha_3T \ \text{ at } r_3\to\infty. \tag{31}$$

Its implementation is possible only with

$$\frac{E_3}{1-2\upsilon_3}C_1^{(3)}-\frac{E_3}{1-2\upsilon_3}\alpha_3T=0. \tag{32}$$

Then

$$C_1^{(3)}=\alpha_3T \tag{33}$$

and

$$\sigma_r^{(3)}(r) = -\frac{2E_3}{r^3(1+\upsilon_3)} C_2^{(3)}. \quad (34)$$

On the basis of Eqs. (23), (30), (33), (34), we reduce Eqs. (23) – (28) to a system of four algebraic equations

$$C_1^{(1)} - C_1^{(2)} - \frac{1}{r_1^3} C_2^{(2)} = 0, \quad (35)$$

$$\frac{E_1}{1-2\upsilon_1} C_1^{(1)} - \frac{E_2}{1-2\upsilon_2} C_1^{(2)} + \frac{2E_2}{r_1^3(1+\upsilon_2)} C_2^{(2)} = \frac{E_1}{1-2\upsilon_1}\alpha_1 T - \frac{E_2}{1-2\upsilon_2}\alpha_2 T, \quad (36)$$

$$C_1^{(2)} + \frac{1}{r_2^3} C_2^{(2)} - \frac{1}{r_2^3} C_2^{(3)} = \alpha_3 T, \quad (37)$$

$$\frac{E_2}{1-2\upsilon_2} C_1^{(2)} - \frac{2E_2}{r_2^3(1+\upsilon_2)} C_2^{(2)} + \frac{2E_3}{r_2^3(1+\upsilon_3)} C_2^{(3)} = \frac{E_2}{1-2\upsilon_2}\alpha_2 T \quad (38)$$

with four unknowns $C_1^{(1)}$, $C_1^{(2)}$, $C_2^{(2)}$, $C_2^{(3)}$.

With the help of Eqs. (35), (37), we express $C_1^{(1)}$ and $C_2^{(3)}$ through $C_1^{(2)}$ and $C_2^{(2)}$

$$C_1^{(1)} = C_1^{(2)} + \frac{1}{r_1^3} C_2^{(2)}, \quad (39)$$

$$C_2^{(3)} = r_2^3 C_1^{(2)} + C_2^{(2)} - r_2^3 \alpha_3 T, \quad (40)$$

and exclude them from Eqs. (36), (38). As a result, we obtain a system of two equations for two unknowns $C_1^{(2)}$, $C_2^{(2)}$

$$\begin{aligned} &\frac{E_1(1-2\upsilon_2) - E_2(1-2\upsilon_1)}{(1-2\upsilon_1)(1-2\upsilon_2)} C_1^{(2)} + \frac{E_1(1+\upsilon_2) + 2E_2(1-2\upsilon_1)}{r_1^3(1-2\upsilon_1)(1+\upsilon_2)} C_2^{(2)} = \frac{E_1\alpha_1 T}{1-2\upsilon_1} - \frac{E_2\alpha_2 T}{1-2\upsilon_2}, \\ &\frac{E_2(1+\upsilon_3) + 2E_3(1-2\upsilon_2)}{(1-2\upsilon_2)(1+\upsilon_3)} C_1^{(2)} + \frac{-2E_2(1+\upsilon_3) + 2E_3(1+\upsilon_2)}{r_2^3(1+\upsilon_2)(1+\upsilon_3)} C_2^{(2)} = \frac{2E_3\alpha_3 T}{1+\upsilon_3} + \frac{E_2\alpha_2 T}{1-2\upsilon_2}. \end{aligned} \quad (41)$$

Let us introduce the notations

$$\begin{aligned} &a_{11} = \frac{E_1(1-2\upsilon_2) - E_2(1-2\upsilon_1)}{(1-2\upsilon_1)(1-2\upsilon_2)}, \quad a_{11} = \frac{E_1(1+\upsilon_2) + 2E_2(1-2\upsilon_1)}{r_1^3(1-2\upsilon_1)(1+\upsilon_2)}, \\ &a_{21} = \frac{E_2(1+\upsilon_3) + 2E_3(1-2\upsilon_2)}{(1-2\upsilon_2)(1+\upsilon_3)}, \quad a_{22} = \frac{-2E_2(1+\upsilon_3) + 2E_3(1+\upsilon_2)}{r_2^3(1+\upsilon_2)(1+\upsilon_3)}, \\ &b_1 = \frac{E_1}{1-2\upsilon_1}\alpha_1 T - \frac{E_2}{1-2\upsilon_2}\alpha_2 T, \quad b_2 = \frac{2E_3}{1+\upsilon_3}\alpha_3 T + \frac{E_2}{1-2\upsilon_2}\alpha_2 T. \end{aligned} \quad (42)$$

Then,

$$C_1^{(2)} = \frac{b_1 a_{22} - a_{12} b_2}{a_{11}a_{22} - a_{12}a_{21}}, \quad C_2^{(2)} = \frac{a_{11} b_2 - b_1 a_{21}}{a_{11}a_{22} - a_{12}a_{21}}. \quad (43)$$

Thus, knowing constants $C_2^{(1)}$, $C_1^{(3)}$ and using Eqs. (42), (43), we can calculate $C_1^{(2)}$ and $C_2^{(2)}$, and then the constants $C_1^{(1)}$, $C_2^{(3)}$ by Eqs. (39), (40). After that, with the help of Eqs. (4) at $i=\overline{1,3}$ all thermal stress functions $\sigma_r^{(i)}(r)$, $\sigma_\varphi^{(i)}(r)$, $\sigma_\theta^{(i)}(r)$ at $i=\overline{1,3}$ are calculated.

As an example, consider the cases where a small-radius $r_2$ rubber inclusion modified in a bituminous medium by chemical treatment and layer 2 of a spherical shell coating with a thickness of $r_2 - r_1 = h$ is formed in it. Space $r_2 < r < r_3$ (at $r \to \infty$) is filled with bituminous medium 3. We assume that the thermomechanical properties of the rubber body 1 $(E_1, \upsilon_1, \alpha_1)$ and bituminous medium 3 $(E_3, \upsilon_3, \alpha_3)$ correspond to the properties of body 1 and medium 2 adopted in section 2 of the article. When setting the thermomechanical characteristics of rubber crumb modified in a bitumen medium, we take into account that they can vary over a wide range depending on the chemical additives used, the temperature and the technology of the modification process. For this reason, when choosing the values $E_2, \upsilon_2, \alpha_2$ of its characteristics, we assume that they can take some intermediate values between the characteristics of rubber and bitumen and set them equal $E_2 = 50$ MPa, while $\upsilon_2$ and $\alpha_2$ are equal to the arithmetic mean values of the quantities $\upsilon_1, \alpha_1$ and $\upsilon_3, \alpha_3$. Then for unmodified rubber $E_1 = 10$ MPa, $\upsilon_1 = 0.49$, $\alpha_1 = 0$, for modified rubber $E_2 = 50$ MPa, $\upsilon_2 = 0.37$, $\alpha_2 = 1.15 \cdot 10^{-4}$ 1/ °C, and for bitumen $E_3 = 3000$ MPa, $\upsilon_3 = 0.25$, $\alpha_3 = 2.3 \cdot 10^{-4}$ 1/ °C. For these values, we consider two cases. In the first case, the system is two-phase, in which the first phase (inclusion 1) is unmodified rubber, and the second is bitumen 3 or modified rubber 2 (phase 1) and bitumen 3 (phase 2). The stress values for these combinations are shown in Table 1.

It follows from this that for the combination $i = 1$, $i = 3$ with rubber inclusion 1, the special properties of rubber associated with its incompressibility ($\upsilon_1 = 0.49$) and zero value $\alpha_1$ are fully manifested. For such system at $T = 30^\circ$ C radial stresses $\sigma_r^{(3)}(r_1)$=3.1245 MPa in bitumen on the contact surface $r = r_1$ exceeded its possible tensile strength $[\sigma]$=3 MPa. This effect may be the cause of the observed in practice effect of the crumbling of rubber inclusions from road surfaces. It should also be noted that in the summertime the temperature of the coating could reach $60^\circ$ C. Obviously, in such situations, the probability of destruction of the coating with rubber inclusion increases even more.

The situation is greatly improved if the rubber crumb is finely dispersed and, due to this, it can be completely modified in bitumen. This case corresponds to the second combination in Table 2. Then the properties of incompressibility and zero thermal expansion of rubber are lost and the maximum values of radial stresses in bitumen drop to $\sigma_r^{(3)}(r_1)$=0.637904 MPa, which is perfectly acceptable. This can explain the increase in the thermal sustainability of the resulting material to sudden temperature changes over time and to its high-gradient inhomogeneity, as well as improved adhesion between different phases in the system [2,8,19,22].

***Table 2*** – Values of thermal stresses in two-phase media

***Таблиця 2*** – Значення термічних напружень у двофазних середовищах

| | $i$ | $E_i$, MPa | $\upsilon_i$ | $\alpha_i$, 1/ °C | $\sigma_r^{(1)} = \sigma_\varphi^{(1)}$, MPa | $\sigma_r^{(3)}(r_1)$, MPa | $\sigma_\varphi^{(3)}(r_1)$, MPa |
|---|---|---|---|---|---|---|---|
| 1 | $i = 1$ | 10 | 0.49 | 0 | 3.1245 | 3.1245 | – 1.5623 |
| | $i = 3$ | 3000 | 0.25 | $2.3 \cdot 10^{-4}$ | | | |
| 2 | $i = 2$ | 50 | 0.37 | $1.15 \cdot 10^{-4}$ | 0.637904 | 0.637904 | – 0.318952 |
| | $i = 3$ | 3000 | 0.25 | $2.3 \cdot 10^{-4}$ | | | |

In the second case under consideration, the system is three-phase, in which phase 1 is unmodified rubber, phase 2 is modified rubber, and phase 3 is bitumen. In this case, the ratio of the smaller radius $r_1$ to a larger radius $r_2$ took on the values $r_1/r_2$ = 0.5 and $r_1/r_2$ = 159/160.

Ratio $r_1/r_2$ = 0.5 corresponds to the incomplete modification of rubber with bitumen, due to which a layer with a thickness of $r_2 - r_1 = r_1$ with intermediate properties $E_2$, $\upsilon_2$, $\alpha_2$ was formed. For comparison, we also consider an example when the dimensions of the rubber inclusion are not very small and therefore it was possible to modify it only to a depth $r_2/160$. Thermal stress values in three fractions $i = 1,2,3$ on contact surfaces $r = r_1$ and $r = r_2$ for the considered situations at $T = 30^\circ$ C are shown in Table 3.

***Table 3*** – Values of thermal stresses in three-phase media

***Таблиця 2*** – Значення температурних напружень у трифазних середовищах

| | $r_1/r_2$ | $\sigma_r^{(1)} = \sigma_\varphi^{(1)}$, MPa | $\sigma_r^{(2)}(r_1)$, MPa | $\sigma_\varphi^{(2)}(r_1)$, MPa | $\sigma_r^{(2)}(r_2)$, MPa | $\sigma_\varphi^{(2)}(r_2)$, MPa | $\sigma_r^{(3)}(r_2)$, MPa | $\sigma_\varphi^{(3)}(r_2)$, MPa |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.5 | 1.1278 | 1.1278 | 0.58266 | 0.80108 | 0.73291 | 0.80108 | – 0.40054 |
| 2 | $\frac{159}{160}$ | 3.0451 | 3.0451 | 2.02604 | 3.0316 | 2.03202 | 3.0316 | – 1.5158 |

Function graphs $\sigma_r(r)$ and $\sigma_\varphi(r)$ for the occasion $r_1/r_2 = 0{,}5$ are given in Fig.5.

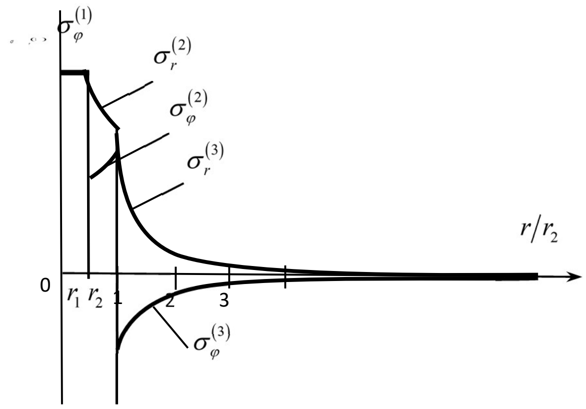


***Figure 5*** – Graphs of changes in thermal stress functions $\sigma_r(r)$ and $\sigma_\varphi(r)$ in a three-phase medium at $r_1/r_2 = 0.5$

***Рисунок 5*** – Графіки зміни функцій термічних напружень $\sigma_r(r)$ і $\sigma_\varphi(r)$ в трифазному середовищі при $r_1/r_2 = 0.5$

A comparison of the thermal stresses given in position 1 of Table 3 and in positions 1 and 2 of Table 2 indicates that the modification of rubber spherical crumb, even in its layer with a thickness equal to half its radius, leads to a significant decrease in radial thermal stresses in bitumen, although to a lesser extent compared to the case of complete modification of the crumb (compare position 1 in Table 3 and position 2 in Table 2).

However, if the size of the crumb is not small, then the modification of its material can be carried out only in a thin near-surface layer. In this case, the decrease in thermal stresses on the surface of its contact with the modified rubber is not significant (compare the data given in position 2 of Table 3 and in position 1 of Table 2). Therefore, plastic deformations can be localized in these zones, leading to the appearance of local defects in bitumen and subsequent chipping of rubber grains.

Thus, it can be concluded that the use of crumb rubber as a filler in asphalt concrete pavement material is associated with additional features conditioned by the thermomechanical properties of rubber.

First, rubber has a very low modulus of elasticity $E$ . In light of this, it weakly resists loads that lead to a change in the shape of the body made from it (for example, uniaxial and shear transport loads).

Second, its Poisson's ratio $\upsilon = 0.48 \div 0.50$, that is, rubber from a technical point of view, is an incompressible material. This property manifests itself as a disadvantage under temperature effects that cause all-round shrinkage or dilation. Since rubber crumb in this case behaves as an absolutely rigid body and prevents free triaxial thermal deformation of bitumen, thermal stress concentrators are generated in it on the contact surface with rubber inclusion.

Thirdly, as shown above, thermal stresses in the fractions of the pavement composite are proportional to the difference in the values of the coefficients of linear thermal expansion of its components. Therefore, it is desirable that these coefficients be of the same sign and have close values. However, for rubber at normal temperatures, the coefficient of linear thermal expansion is close to zero or negative. This factor further contributes to the growth of thermal stresses in the contact zone of the rubber inclusion and bitumen. The results of the calculations allow us to quantitatively confirm the conclusions obtained in practice about the effectiveness of the procedure for modifying crumb rubber with bitumen and use it to further improve this material.

In addition, it should be noted that the studies carried out were performed using the model of spherical rubber crumbs. In reality, these crumbs may have irregular faceted shapes with ribs and vertices, which play the role of additional thermal stress concentrators. This factor must be taken into account when analyzing the thermomechanics of the materials under consideration.

**Conclusions**

1. Based on the methods of thermoelasticity, the problem of the thermally stressed state of an asphalt concrete pavement with rubber inclusions was posed. For a spherical model of a homogeneous inclusion and an inclusion covered with a spherical layer of modified rubber, the resolving equations for the thermoelastic deformation of the system are formed, and their analytical solutions are constructed.

2. As a result of the analysis of the obtained solutions, an explanation is given for the features of the thermally stressed behavior of the combined system, which consists in the possibility of maintaining the integrity of the system with finely dispersed rubber grains and chipping of relatively large rubber inclusions.

3. When analyzing the features of the thermomechanical behavior of an asphalt concrete material with rubber inclusions, attention was drawn to the specifics of the thermomechanical properties of rubber associated with a low value of its elastic modulus, a large Poisson's ratio (which makes rubber a practically incompressible material) and a small value of the coefficient of linear thermal expansion (zero or even negative at ordinary temperatures). It is shown that in connection with this properties of rubber in bitumen in the zone of its contact with inclusions, additional concentrations of thermal stresses can occur, caused by thermomechanical incompatibility of the phases of the composite and leading to the initiation of additional latent plastic strains and destructions in bitumen on the contact surface, causing chipping of inclusions.

4. Using analytical calculations, it has been demonstrated that the mitigation of additional thermomechanical defects and their elimination can be achieved by reducing the thermomechanical incompatibility of the properties of the composite phases by bituminous modification of rubber. As this takes

place, the efficiency of this method increases with an increase in the depth of penetration of the modified layer into the rubber grain.

## References

1. Abbas Mohajerani, Lucas Buznett, John V. Smith, Stefan Markovski, Glen Rodwell, Md Tareq Rahman, Halenur Kurmus, Mehdi Mirzbabaei, Arul Arulrajan, Suksun Horpibulsuk, Farsid Maghool. Recycling waste rubber tyres in construction materials and associated environmental considerations: A review. Resources, Conservation and Recycling, Vol. 155, April 2020, 104679, pp.1-17 https://doi.org/10.1016/j.resconrec.2020.104679

2. Delatte N. Concrete Pavement Design, Construction, and Perfomance. Taylor & Francis, London and New York, 2008, 1-372.

3. Krishnan J.M., Rajagopal K.R. Review of the uses and modeling of bitumen from ancient to modern times. American society of mechanical engineers. Appl Mech Rev, 2003, 56(2), P. 149–214.

4. Nuha Salim Mashaan, Asim Hasan Ali, Mohamed Reham Karim, Mahrez Abdelaziz. An overview of crumb rubber modified asphalt. Intern. Journ. Of Physical sciences, 2011, 7(2), 166-170. Doi:105897/IJPSX11.007.

5. Paravita Sri Wulandari, Daniel Tjandra. Use of crumb rubber as an additive in asphalt concrete mixture. Procedia Engineering. 2017, V.171, pp.1384-01389.

6. Presti D.L. Recycled tyre rubber modified bitumens for road asphalt mixture: a literature review. Construction and Building Materials. 2013, 49, pp.863-881.

7. Rokade S. Use of waste plastic and waste rubber tyres in flexible highway pavements. International Conference on Future Environment and Energy IPCBEE. 28, 2012, pp. 105-108.

8. Marcela Fiedlerova, Petr Jisa, Kamil Stepanek. Using various thermal analytical methods for bitumen characterization. International Journal of Pavement Research and Technology, 2021, V.14, Issue 4, pp. 459-465.

9. Tao Ma, Yongqiang Zhong, Tao Tang, Xiaoming Huang. Design and evalution of heat-resistant asphalt mixture for permafrost regions, Internationl Journal of Civil Engineering, 2016, V.14, 5. Doi:10.1007/s40999-16-0039-9.

10. Zhang D., Huang X., Zhao Y., Zhang S. Rubberized asphalt mixture design using a theoretical model. Construction and Building Materials. V.67, 2014, pp.265-269.

11. Butz T., Muller J., Riebesehl G. Innovative method for producing crumbed rubber modified asphalt. 5-th Eurasphalt & Eurobitume Congress. Istanbul (2012).

12. Gao W. Study on properties of recycled tyre rubber modified asphalt mixture using dry process. Construction and Building Materials. 2007, 21, pp.1011-1015.

13. Mashaan N.S., Ali A.H., Karim M.R., Abdelaziz M. Effects of crumb rubber concentration on physical and rheological properties of rubberised bitumen binders. International Journal of Physical Sciences. 2011, V.6,4. Pp.684-690.

14. Raheb Mirzanamadi, Par Johansson, Sotirios Grammaticos. Thermal properties of asphalt concrete: A numerical and experimental study. Construction and Building Materials. 2018, V.158, pp. 774-785.

15. Christensen R.M. Mechanics of Composite Materials; Wiley: New York, NY, USA, 1979, 348 p.

16. Leslie R.G. Treloar. The mechanics of rubber elasticity. Journal of Polymer Science Polymer Symposia, March 2007, 48(1), pp. 107-123. https://doi.org/10.1002/polc.5070480110

17. Lyon R.E., Farris R. J. Thermomechanics of rubber at small strains. Polymer. 1987, V.28, 7, pp.1127-1132.

18. Miller W., Smith S.W., Mackenzie D.S., Ewans K.E., Negative Thermal Expansion: a Review Journal of Material Science, Vol. 44, pp.5441–5451, 2009.

19. Takenaka K., Negative Thermal Expansion Materials: Technological Key for Control of Thermal Expansion, Science and Technology of Advanced Materials, Vol. 13, pp.1-11, 2012

20. Khazanovich L. The elastic-viscoelastic correspondence principle for non-homogeneous materials with time translation non-variant properties. Intern. Journ. Solids and Structures. 2008, Vol. 45,15, pp.1-9.

21. Paulino G.H., Jin Z.-H. Correspondence principle in viscoelastic functionally graded materials. ASME J. Appl. Mech. 2001, Vol.68, pp. 129-132.

22. Carlson D.E., Thermoelasticity, Encyclopedia of Physics (ed. Trusdell C., Springer, Berlin, 1972).

23. Kovalenko, A.D. Osnovy termopruzhnosti [Thermoelasticity fundamentals]. Kyiv: Naukova Dumka, 1970. 307 s. [in Russian].

24. Noda N., Hetnarski R.B., Tanigawa Y., Thermal Stresses (2nd edition, New York: Taylor and Francis, 2003).

25. Nowacki W. Thermoelasticity, 2 nd ed. Oxford: PWN – Polish Scientific Publishers, Warsaw and Pergamon Press, 1986.

## ТЕОРЕТИЧНЕ МОДЕЛЮВАННЯ ЗАРОДЖЕННЯ ВНУТРІШНІХ ПРИХОВАНИХ ТЕРМІЧНИХ ДЕФЕКТІВ У БІТУМНОМУ СЕРЕДОВИЩІ З ГУМОВИМИ ВКЛЮЧЕННЯМИ

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**Анотація.** Сучасний стан і прогрес технології та науки, пов’язані з повторним використанням та переробкою шинної гуми в дорожній промисловості в усьому світі, змушує більш ретельно вивчати високо- та низькотемпературні характеристики дорожнього бітуму, модифікованого гумовою крихтою, що дозволяє зрозуміти вплив температури, зернистість гуми та модифікації суміші бітум-каучук на міцність і стійкість композиту. Нижче ці питання вивчаються з урахуванням особливостей термомеханічних властивостей гуми, пов'язаних з її малою жорсткістю при зміні форми, практичною нестисливістю при зміні об'єму та малим (нульовим або навіть від’ємним) коефіцієнтом лінійного температурного розширення.

Метою дослідження є визначення причин, що проводять до порушення міцності асфальтобетонних матеріалів з домішками гумових крихт. Для цього аналізується вплив несумісності термомеханічних характеристик (модулів пружності, коефіцієнтів Пуассона та коефіцієнтів температурного розширення) бітуму та гуми на концентрацію додаткових внутрішніх термонапружень в системі, що викликані сезонними та добовими змінами температури.

На основі співвідношень теорії термопружності побудовано математичну модель термодеформування гумової крихти в бітумному середовищі. З можливістю повного та поверхневого модифікування гуми бітумом побудовано розв'язки для трифазних середовищ, що дозволяють простежити вплив параметрів кожної фази на поля термонапружень в системі. Встановлено, що додаткові термонапруження в бітумі, зумовлені термомеханічною несумісністю фізичних параметрів фаз, концентруються в зоні контакту з поверхнею гумової крихти і можуть бути причиною виникнення в ній дефектів, а потім і викришування крихти. Розглянуто вплив ефекту модифікації гумової крихти бітумом та глибини проникнення її у крихту різних розмірів на зниження термонапружень у системі й підвищення її стійкості.

**Ключові слова:** асфальтобетонний матеріал, гумові включення, нестисливість гуми, концентратори термонапружень, модифікована гума.

### Перелік посилань

1. Abbas Mohajerani, Lucas Buznett, John V. Smith, Stefan Markovski, Glen Rodwell, Md Tareq Rahman, Halenur Kurmus, Mehdi Mirzbabaei, Arul Arulrajan, Suksun Horpibulsuk, Farsid Maghool. Recycling waste rubber tyres in construction materials and associated environmental considerations: A review. Resources, Conservation and Recycling, Vol. 155, April 2020, 104679, pp.1-17 https://doi.org/10.1016/j.resconrec.2020.104679

2. Delatte N. Concrete Pavement Design, Construction, and Perfomance. Taylor & Francis, London and New York, 2008, 1-372.

3. Krishnan J.M., Rajagopal K.R. Review of the uses and modeling of bitumen from ancient to modern times. American society of mechanical engineers. Appl Mech Rev, 2003, 56(2), P. 149–214.

4. Nuha Salim Mashaan, Asim Hasan Ali, Mohamed Reham Karim, Mahrez Abdelaziz. An overview of crumb rubber modified asphalt. Intern. Journ. Of Physical sciences, 2011, 7(2), 166-170. Doi:105897/IJPSX11.007.

5. Paravita Sri Wulandari, Daniel Tjandra. Use of crumb rubber as an additive in asphalt concrete mixture. Procedia Engineering. 2017, V.171, pp.1384-01389.

6. Presti D.L. Recycled tyre rubber modified bitumens for road asphalt mixture: a literature review. Construction and Building Materials. 2013, 49, pp.863-881.

7. Rokade S. Use of waste plastic and waste rubber tyres in flexible highway pavements. International Conference on Future Environment and Energy IPCBEE. 28, 2012, pp. 105-108.

8. Marcela Fiedlerova, Petr Jisa, Kamil Stepanek. Using various thermal analytical methods for bitumen characterization. International Journal of Pavement Research and Technology, 2021, V.14, Issue 4, pp. 459-465.

9. Tao Ma, Yongqiang Zhong, Tao Tang, Xiaoming Huang. Design and evalution of heat-resistant asphalt mixture for permafrost regions, Internationl Journal of Civil Engineering, 2016, V.14, 5. Doi:10.1007/s40999-16-0039-9.

10. Zhang D., Huang X., Zhao Y., Zhang S. Rubberized asphalt mixture design using a theoretical model. Construction and Building Materials. V.67, 2014, pp.265-269.

11. Butz T., Muller J., Riebesehl G. Innovative method for producing crumbed rubber modified asphalt. 5-th Eurasphalt & Eurobitume Congress. Istanbul (2012).

12. Gao W. Study on properties of recycled tyre rubber modified asphalt mixture using dry process. Construction and Building Materials. 2007, 21, pp.1011-1015.

13. Mashaan N.S., Ali A.H., Karim M.R., Abdelaziz M. Effects of crumb rubber concentration on physical and rheological properties of rubberised bitumen binders. International Journal of Physical Sciences. 2011, V.6,4. Pp.684-690.

14. Raheb Mirzanamadi, Par Johansson, Sotirios Grammaticos. Thermal properties of asphalt concrete: A numerical and experimental study. Construction and Building Materials. 2018, V.158, pp. 774-785.

15. Christensen R.M. Mechanics of Composite Materials; Wiley: New York, NY, USA, 1979, 348 p.

16. Leslie R.G. Treloar. The mechanics of rubber elasticity. Journal of Polymer Science Polymer Symposia, March 2007, 48(1), pp. 107-123. https://doi.org/10.1002/polc.5070480110

17. Lyon R.E., Farris R. J. Thermomechanics of rubber at small strains. Polymer. 1987, V.28, 7, pp.1127-1132.

18. Miller W., Smith S.W., Mackenzie D.S., Ewans K.E., Negative Thermal Expansion: a Review Journal of Material Science, Vol. 44, pp.5441–5451, 2009.

19. Takenaka K., Negative Thermal Expansion Materials: Technological Key for Control of Thermal Expansion, Science and Technology of Advanced Materials, Vol. 13, pp.1-11, 2012

20. Khazanovich L. The elastic-viscoelastic correspondence principle for non-homogeneous materials with time translation non-variant properties. Intern. Journ. Solids and Structures. 2008, Vol. 45,15, pp.1-9.

21. Paulino G.H., Jin Z.-H. Correspondence principle in viscoelastic functionally graded materials. ASME J. Appl. Mech. 2001, Vol.68, pp. 129-132.

22. Carlson D.E., Thermoelasticity, Encyclopedia of Physics (ed. Trusdell C., Springer, Berlin, 1972).

23. Kovalenko, A.D. Osnovy termopruzhnosti [Thermoelasticity fundamentals]. Kyiv: Naukova Dumka, 1970. 307 s. [in Russian].

24. Noda N., Hetnarski R.B., Tanigawa Y., Thermal Stresses (2nd edition, New York: Taylor and Francis, 2003).

25. Nowacki W. Thermoelasticity, 2 nd ed. Oxford: PWN – Polish Scientific Publishers, Warsaw and Pergamon Press, 1986.

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